Effects of input representation on self-organizing and bottom-up model of chord schema

Nart Bedin Atalay*

Abstract
Effects of input representation on self-organizing and bottom-up model of chord schema were investigated. A single layer self-organizing map was trained with 12 major and 12 minor chords. Training was repeated five times, each time with a different input representation. In this model, activation of chords is determined by the representation and activation of pitches that compose the chords. Important findings of chord perception were simulated with this model. Simulation results showed that input representation was critical to simulate all of the findings.

Keywords: chord perception; self-organizing maps; bottom-up models; chord schema; cognitive model

Giriş gösterimlerinin kendini örgütleyen ve aşağıdan yukarıya çalışan akor şemalarına etkisi

Nart Bedin Atalay*

Özet

Anahtar Kelimeler: akor algoritma; kendini örgütleyen ağlar; aşağıdan yukarıya çalışan modeller; akor şemaları; bilişsel model

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Introduction
Since the inception of the cognitivist approach to the musical mind, numerous symbolic AI and connectionist models of music perception have been proposed (see Balaban, Ebcioglu & Laske, 1992; Bharucha, 1988; Griffith & Todd, 1994; Todd & Loy, 1991; see also Toiviainen, 2000 for a critical review). Some of them were introduced as working hypotheses about the architecture and mechanism of the components of the musical mind. These models have been developed hand-in-hand with the experimental studies: results of experiments on music cognition have been compared with outputs of the models; matched/mismatched comparisons have been the motivation to vindicate/reject the proposed architecture and/or mechanism.

Many aspects of music perception have been modeled (see Purwins, Grachten, Herrera, Hazan, Marxer, & Serra, 2008; Purwins, Herrera, Grachten, Hazan, Marxer, & Serra, 2008 for reviews). In this study, the focus is on modeling chord perception with neural networks. The classic neural network model of chord perception is MUSACT (MUSical ACTivations, Bharucha, 1987). MUSACT is an interactive theory of tonal schema. According to the interactive view, mental representation of a chord is determined both by the mental representation of pitches of the chord, and the mental representation of the key of the musical context in which the chord resides. MUSACT (Bharucha, 1987) proposes a hierarchical organization of mental representations of pitches, chords, and keys, which occupy different layers in the neural network (see Figure 2). Information flow between layers is bi-directional (bottom-up and top-down). The activation flows from the lower layers to upper layers (bottom-up activation) as well as from upper-layers to lower layers (top-down activation). For example, activation of the chord layer is shaped both by the pitch layer (the bottom-up effect) and key layer (the top-down effect). MUSACT (Bharucha, 1987) has successfully simulated effects of musical context on chord perception (see Krumhansl, 2005; Tillman, Bharucha & Bigand, 2000 for review).

Connections of MUSACT were hand-wired (Bharucha, 1987); they were not developed as a result of training the network with a representative sample of Western tonal music. This is not a common method for modeling perceptual processes, especially when they depend on learning (see Rumelhart and McClelland, 1986). A complete model of chord perception “… should simulate the internalization of Western pitch regularities by mere exposure, allowing
the connection weights to adapt to the musical environment.” (Tillmann et al. 2000, p. 891). Tillmann et al. (2000) showed that this was possible.

Tillmann et al. (2000) used a two-layer self-organizing map (Kohonen, 1982). Layers were trained one after the other, with simultaneous and sequential combinations of tones. Consequently, the first layer represented the major and minor chords, and the second layer represented major tonalities. Then, Tillmann et al. (2000) pruned some connections and scaled weights between layers. Pruning and scaling of connections were necessary to build a working top-down and bottom-up signal flow between layers. At the end, the two-layer map was architecturally equivalent to MUSACT (Bharucha, 1987), i.e. layers were organized hierarchically, and information flow was bi-directional (top-down and bottom-up). With this model, Tillmann et al. (2000) simulated a broad range of experiments that investigated perception of tones, chords and keys.

This study is a follow-up of Tillmann et al. (2000). The goal is to develop a bottom-up model of chord schema with self-organization. A single-layer self-organizing map was trained with chord categories; and there was no pruning and scaling of connections. The model has bottom-up connections (no top-down connections) operate on a single layer. In addition, effects of pitch representation on the performance of the model were investigated.

The article is organized as follows. The first section briefly summarizes important findings on the perception of chords. The second section introduces the MUSACT (Bharucha, 1987) model. The third section informed the reader about the mechanism of self-organizing maps; and the fourth section describes self-organizing models of tonal schema, especially of Tillmann et al. (2000). The fifth section reports a bottom-up self-organizing model of chord schema. The last section reports and discusses the results of simulations.

**Perception of chords**

It is worth to note that, the study of perception of chords is not limited to those summarized below (see Justus & Bharucha, 2002; Krumhansl, 1990, 1991, 2000, 2005 for comprehensive reviews). In this section, important findings on the perception of chords are summarized. Perception of chords has been investigated with chord similarity ratings (Bharucha & Krumhansl, 1983; Krumhansl, Bharucha, Kessler, 1982), recognition memory for chord
sequences (Bharucha & Krumhansl, 1983), chord priming (Bharucha & Stoeckig, 1986), and probe chord paradigms (Krumhansl 1990).

Probe chord paradigm investigates the perceptual fit of a chord into a tonal context. In Krumhansl (1990), participants listened to a musical context, followed by a probe chord. The context was a scale or a chord sequence of a major or minor key. Participants were asked to rate how well the probe chord fit with the context (Krumhansl, 1990). The ratings of chords were hierarchical: the highest score was given to tonic chord, followed by subdominant, dominant, and mediant, followed by other diatonic chords, followed by non-diatonic chords (see Krumhansl, 1990, Table 7.3).

Chord similarity experiments (Krumhansl et al., 1982; Bhaucha & Krumhansl, 1983) investigate how well two chords sound together after a musical context. In Bhaucha and Krumhansl (1983), participants listened to chord sequences in five different keys (C, G, A, B, and F# major) followed by all possible pairs of chords from the C and F# major keys. Results showed that the higher the two chords were in the hierarchy, the similar they were. This was called contextual distance principle (Bhaucha & Krumhansl, 1983). For example, C and G major chords were perceived more similar when they followed C major key (C and G major chords are tonic and dominant of the C major key.), compared to when they followed F# major key (C and G major chords are non-diatonic chords of the F# major key.). In addition, results showed that the last chord of the pair contributed the perceived similarity more, compared to the first one. This was called contextual asymmetry principle (Bhaucha & Krumhansl, 1983). For example, C and F# major chords were perceived more similar when they followed F# major key (F# major chord is tonic of the F# major key) compared to when they followed C major key (F# major chord is a non-diatonic chord of the F# major key). Krumhansl (1990) in Table 8.2 reproduced ratings given to diatonic chord pairs observed in Bhaucha and Krumhansl (1983). Analyses showed that contextual distance and contextual asymmetry principles apply to the ratings of diatonic chords as well (Krumhansl, 1990).

In chord priming paradigm, effects of tonal context on the processing of the target chord are examined. The tonal context (prime) may be a single chord (Bharucha & Stoeckig, 1986) or a chord sequence (Bigand & Pineau, 1997). The target chord follows the context immediately. Participants are asked to make a binary judgment with an objectively correct

In Bharucha and Stoeckig (1986), participants listened to two chords; and they were asked to decide whether the second chord was in-tune or not. Results showed that participants were faster and more accurate for their judgments when the priming chord was harmonically related to the target chord compared to when they were not related. (For example, C major is primed more by Bb major chord compared to F# major.) This was true even if the harmonically unrelated target share a pitch with the prime (Tekman & Bharucha, 1998). (For example, C and E major chords share a pitch, but C major is primed more by D major chord compared to E major.) In other words, chord priming is not governed by acoustical similarity (Tekman & Bharucha, 1998).

In Bigand and Pineau (1997), participants listened to two chord sequences; and they were asked to decide whether the last chord of the sequence was consonant or not. The two chord sequences were arranged in different keys. The last two chords of the sequences were identical. Depending on the key of the sequence the final chord (the target) was either tonic or subdominant. Participants were faster and more accurate for their judgments when the target was tonic compared to when it was subdominant (Bigand & Pineau 1997, Bigand, Madurell, Tillmann & Pineau 1999, Experiment 1). This effect was called global priming. The penultimate chord (the chord that immediately preceded the target) was identical in different sequences. Therefore, factors that may cause priming could not be local (within the immediate vicinity of the target).

The amount of musical training and the explicit knowledge about music did not correlate with chord priming (Bharucha & Stoeckig, 1987; Tekman & Bharucha, 1992, 1998); and global priming of chords has been repeatedly observed from both musicians and non-musicians (Bigand & Pineau 1997; Bigand et al., 1999; Bigand, Poulin, Tillmann, & D’Adamo, 2003). These results demonstrate that chord priming is a result of an implicit musical learning process.
Circle of fifths (Figure 1) is a geometric representation of harmonic relation between chords. Chords closer on the circle of fifths are harmonically related. There has been one-to-one correspondence between the results of harmonic priming studies and the distance on the circle of fifths: primes (both local and global) closer to the target on the circle of fifths facilitated responses compared to distant primes (Bharucha & Stoeckig, 1986, 1987; Bigand & Pineau, 1997; Bigand et al. 1999; Justus & Bharucha, 2002; Tekman & Bharucha, 1992, 1998). In other words, there is a close link between harmonic relation and harmonic priming.

Bharucha (1987) proposed that chord priming is due to the implicit knowledge about the relationship between pitches, chords and tonalities. He implemented the implicit tonal knowledge with a neural network model, which is called MUSACT (Bharucha, 1987). The MUSACT model (Bharucha, 1987) simulated results of all local chord priming experiments (Bharucha & Stoeckig, 1987; Justus & Bharucha, 2002; Tekman & Bharucha, 1992, 1998) and many global chord priming experiments (Bigand & Pineau 1997, Bigand et al. 1999, Experiment 1; Bigand et al., 2003; Tillmann, et al.,1998; Tillmann, Janata, Birk, & Bharucha, 2003). Bharucha (1992) and Tillmann et al. (2000) argued that feature detectors that are sensitive to statistical properties of events in the environment could learn implicit tonal harmonic knowledge.
MUSACT

MUSACT (Bharucha, 1987) is an artificial neural network consisting of three groups of interconnected units, which represents pitch categories (12 units), major and minor chords (12 units for each), and major keys (12 units) (Figure 2). Every chord unit is connected to the units representing the three pitches contained in that chord. Every major key unit is connected to the units representing the three major and three minor chords that are possible in that key. All connections are bi-directional. Connections and connection weights of MUSACT are hand-wired.

![Figure 2. MUSACT (Bharucha, 1987).](image)

Functioning of the network is based on phasic spreading of activation. Each unit has a level of activation associated with it. Only the activation of the pitch units can be changed by an external input. All other changes in the activation result from the activation transmitted through the weighted connections and the decay of activation towards zero. Transmission of activation from a unit is proportional to the difference between what the unit holds and receives as a signal. The spread of activation continues until the equilibrium is reached. Equilibrium is defined as an activation change below a threshold value.

Tillmann et al. (2000) have demonstrated that implicit tonal harmonic knowledge could develop with self-organizing maps as a result of training with combinations of tones and combinations of chords that are typical in Western tonal music. Their model highly correlated with MUSACT (Bharucha, 1987). Tillmann et al. (2000) gave accounts of various empirical studies on perception of tones, chords and keys with their model.
Self-organizing maps

Self-organizing map is a topologically correct feature map that develops with respect to input signals (Kohonen, 1982). It is a feature map because units of the map become sensitive to different features of the input signal after training. It is self-organizing because the training algorithm works in an unsupervised way; i.e. during training there is no teacher to depict features and the corresponding signals. It is a map because units are located on a surface, and they have specific distance relation determined by the surface. It is a topologically correct feature map because nearby units on the map become sensitive to similar features. Self-organizing maps are designed in such a way that "... external signal activity alone ... is sufficient for enforcing mappings ... into the system." (Kohonen, 1982, p.59).

The training algorithm of self-organizing map is simple. Training signals are presented to the map one by one in random order. For each signal the best matching unit, whose weight vector is closest to the signal vector, is detected (Formula 1). Units that lie within the neighborhood of the best matching unit are subject to weight update. The weight update function rotates the weight vector towards the direction of the signal vector insignificantly (Formula 2). The amount of rotation is a function of the learning rate (Formula 3) and the distance between the best matching unit and the unit whose weight is updated (Formula 4).

Formulae presented below are from Vesanto, Himberg, Alhoniemi, Parhankangas (2000). i is the input vector and $w_j$ is the weight vector of $j$th unit. $|a|$ refers to the length of vector a. Weight vector of best matching unit is marked with *.

Formula of best matching unit is

$$|w_{j*} - i| \leq |w_j - i| \quad \forall j$$  \hspace{1cm} (1)

Formula of weight update is

$$w_j(t + 1) = w_j(t) + \alpha(t) h_{ij}(t) [i(t) - w_j(t)]$$  \hspace{1cm} (2)

where learning rate is

$$\alpha(t) = \alpha_0 (0.005 / \alpha_0)^{t/T}$$  \hspace{1cm} (3)
and unit distance (neighborhood) is

\[ h_{cj}(t) = e^{-d_{cj}^2 / 2\sigma_t^2} \]  \hspace{1cm} (4)

During training learning rate and neighborhood gradually decrease. Training stops when the learning rate is zero. Kohonen (2000) suggests that better map organization is achieved by splitting training into two phases: rough organization and fine tuning. In the rough organization phase, neighborhood is large, learning rate is high and training takes short. In the fine tuning phase, neighborhood and learning rate are small and training takes long.

Self-organizing maps (Kohonen, 1982) produce topologically correct maps with neuron-like computations. Topological mappings were found in many places of the brain (Knudsen, du Lac, & Esterly, 1987 cited in Kohonen, 1982), which suggests that it has an important function for neural computation. Tonotopic mapping is an example of topological mapping. In tonotopic maps, spatial arrangement of neurons reflects frequencies of sound waves. One of the tonotopic maps of the nervous system is located in the right-temporal cortex (Pantev, Hoke, Lehnertz, Lütkenhöner, Anogianakis, & Wittkowski, 1988).

**Self-organizing models of chord schema**

MUSACT (Bharucha, 1987) is a neural network model of tonal schema. Units, connections and connection weights of MUSACT were hand-wired (Bharucha, 1987). However connections and weights of the model of tonal schema should develop by a self-organizing mechanism that adapts regularities in the musical environment (Tillmann et al. 2000, p. 891).

Tillmann et al. (2000) showed that this was possible by training two self-organizing maps that are connected in layers. The first map was composed 6X6 (36) units located on a square surface. It was trained with chord categories to develop representations of chords. Training data were composed of 24 vectors, which represented 12 major and 12 minor chord categories. Chords were coded with either sparse or rich coding. In sparse coding, chords were represented by three pitches (pitches of the chord), whose weights were equal. Chords that share pitches were similar to each other in sparse coding. In rich coding chords were represented by the sum of the subharmonics of the pitches of the chords (Parncutt, 1988). Rich encoding mirrored similarities between psycho-acoustical representations of chords.
(Leman, 1995; Terhardt, 1974; Parnicut, 1988). At the end of training, 24 units of the first map represented 12 major and 12 minor chord categories. These units were located on the first map with respect to the similarity between input representations of different chords (see Figures 4 and 6 of Tillmann et al., 2000).

The second map was composed of 4X4 (16) units, which were fully connected to the units of the first map. The second map was trained with chord sequences to develop representations of keys. This was done after the first map was fully trained. Simulations were carried with two chord corpora. In the first corpus (the small corpus) there were 12 chord sequences, which were composed of six chords. The chords were the three major and the three minor chords that were possible in a major key. In the second corpus (the large corpus) there were 120 chord sequences. Ten chord sequences were constructed semi-randomly by referring to common chord progressions (Piston, 1978), statistical distributions of chords (Budge, 1943, cited in Tillmann et al., 2000) and termination rules of chord sequences (Piston, 1978). These sequences were played in 12 major keys making 120 chord sequences in total. Pitches that composed chords of the sequences were presented to the first map. Activation of the best matching unit on the first map was kept constant till the end of the sequence. The best matching unit was always one of the 24 chord categories. The second map was trained with the activation pattern of the first map observed at the end of sequences. After training, 12 units of the second map represented 12 major keys. These units were located on the second map by forming a circle of fifths. As a result, hierarchically self-organized maps detected chord and tonality categories at this stage of training.

These maps were not architecturally identical to MUSACT (Bharucha, 1987). MUSACT has bi-directional connections. Activation reverberates between layers in MUSACT; and after some time the reverberation reaches a steady state. The map of Tillmann et al. (2000) had only bottom-up connections at this stage. In order to convert the architecture of the two-layer self-organized map to the architecture of MUSACT (Bharucha, 1987), Tillmann et al. (2000) pruned some links and scaled weights. Pruning was necessary to rule out reverberation of activation between units that did not represent a category. At the pruning step, training stimuli were presented again to the network. For each stimulus, the unit with the highest activation was detected. Weights of links between the input signal and best matching unit were increased and the remaining weights were decreased (Tillmann et al., 2000). At the end
of pruning, only the three pitches contained in a chord are connected to the unit that represents the chord; and only the three major and three minor chords that are possible in a key are connected to the unit that represents the key. In other words, after pruning, the linkage between units became identical to the MUSACT (Bharucha, 1987) model. Tillmann et al. (2000) scaled weights of the remaining links. Scaling of weights was necessary to ensure the termination of reverberation of activation after some time. The termination of reverberation depends on eigenvalues of connection matrices, which must be lower than 1 (footnote 7, Tillmann et al. 2000). In order to meet this constraint, weights of the network were scaled (Tillmann et al. 2000, p. 894). It is worth to note that, pruning and scaling are not part the self-organizing map algorithm.

Tillmann et al. (2000) simulated wide range of experimental results on perception chords. Tillmann et al. (2000) showed that principles of chord perception, i.e. contextual identity, contextual distance, and contextual asymmetry (Bharucha & Krumhansl, 1983) were simulated with their model. They simulated the local and global chord priming effects (Bharucha & Stoeckig, 1986; Tekman & Bharucha, 1998; Bigand & Pineau, 1997); and ERP findings that used chords as stimuli (Patel, Gibson, Ratner, Besson, & Holcomb, 1998).

Activations of the self-organized chord schemata of Tillmann et al. (2000) were highly correlated with activations of MUSACT (Bharucha, 1983). The lowest correlation was observed with the model that was trained with the rich coding and the big corpus (r=0.97, p<0.01, N=24) (Tillmann et al., 2000, Table 1). The highest correlation observed with the model that was trained with the sparse coding and the small corpus (r=0.99, p<0.01, N=24) (Tillmann et al., 2000, Table 1).

Although simulations carried by Tillmann et al. (2000) were quite successful, steps taken on the way to develop their model depended on many questionable assumptions. According to the self-organization scenario, chords are learned earlier than keys; and it is necessary to develop a chordal representation to learn keys. These hypotheses were not supported by empirical findings in Tillmann et al. (2000). Similarly, pruning and scaling steps suggested corresponding stages in the time-course of learning chord and tonality categories. Most

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1 They simulated findings on perception of pitches and keys as well.
importantly, activation and output algorithm of a unit in Tillmann et al. (2000) changed at the time of training and testing the network. At the time of training they followed the procedures of self-organizing maps (Kohonen, 1982); at the time of training they were identical to MUSACT (Bharucha, 1987). Activation and output functions of MUSACT and self-organizing map are different. Activation of a unit on MUSACT is addition of three elements: activation directly coming from input, previous activation, and activation as a result of the reverberation in the network (Tillmann et al. 2000, Formula 1). On the other hand, activation of a unit on a self-organization map depends only on the current input (Tillmann et al. 2000, Formula 2; Kohonen, 1982, Formula 1). Output of a unit on MUSACT is the difference between signal that unit holds and receives (Bharucha, 1987). On the other hand, output of a unit on self-organizing map is a linear function of input (i.e. input and output was identical) (see Tillmann et al. 2000, Formula 2). Tillmann et al. (2000) did not explain these changes between the learning stage and the final-stage of the network.

Tillmann et al. (2000) was not the only study that investigated self-organization of chord schema. Leman (1995) and Leman and Carreras (1997) trained self-organizing maps (Kohonen, 1982) to model learning of chords and keys. Their maps were composed of 20X20 (400) units (Leman, 1995) and 100X100 (10000) units (Leman & Carreras, 1997) located on a torus. Input representations were firing patterns of auditory nerves calculated from acoustical signals. Calculations were based on models of perception of pitches and chords (Parncutt, 1988; Terhadt, 1982; Terhardt, Stoll & Seewan, 1982a,b; Van Immerseel & Martens, 1992), each of which generated a different input representation. In some simulations temporal relations between inputs were represented with a short-term memory model. Short-term memory model integrated activations of recent history into the current activation pattern (Leman, 1995, p.100).

Leman (1995) utilized two training sets. The first training set consisted of 115 chord categories. (These were 12 major, 12 major, 12 diminished, 4 augmented, 12 major seventh, 12 minor seventh, 12 dominant seventh, 12 half-diminished seventh, 12 augmented-seventh, 12 minor with major seventh, 3 diminished seventh chords.) Acoustical representations of chord categories were converted to the input representations (the patterns of auditory nerve firings). The second training set consisted of 48 chord sequences. Sequences were divided into 72 acoustical windows. Acoustical representations of each window were converted to
the input representations, on which effects of short-term memory were added. Leman and Carreras (1997) utilized real performances of Johann Sebastian Bach’s Well-Tempered Clavier as the training set. Training stimuli were divided into acoustical windows of three seconds, and converted to the patterns of auditory nerve firings.

Training different network structures with different sets of data gave similar results (see Leman, 1995; Leman & Carreras, 1997). After training, units represented chord and tonality categories as a function of the training set. Tonality category units were located in a way that reminded the circle of fifths. Location and activation of chord category units reflected the organization of the circle of fifths. On the other hand, Leman (1995) and Leman and Carreras (1997) did not simulate perception of chords with their models.

Gjerdingen (1992) employed the ART 3 architecture (Carpenter & Grossberg, 1990) to train a self-organizing network sensitive to temporal patterns of chords. A short-term memory was employed to convert temporal patterns of chords into activation patterns of input. However, this model was not designed to capture the perception of chords (Gjerdingen, 1992, p. 563).

**Bottom-Up Self-Organizing Model of Chord Schema**

The goal of this set of simulations is to show that findings on chord perception can also be simulated with self-organizing networks that have only bottom-up connections. This study is a follow-up of Tillmann et al. (2000). They trained a two-layer self-organizing map to develop a tonal-schema model that contains bottom-up and top-down connections. Tillmann et al. (2000) simulated various findings on perception of chord, key and pitch with their model. On the other hand, in order to guarantee the correct operation of top-down connections, they pruned and scaled connections, and changed the activation and output algorithms of the units in the network. In this study, the aim is to develop self-organizing model of chord schema that is functionally, but not structurally, equivalent to MUSACT (Bharucha, 1987). Unlike Tillmann et al. (2000), there were no computations apart from those described in Kohonen (1982), i.e. no pruning and scaling of weights; no change in the output and activation functions. In addition, effects of input representation on the performance of the model were investigated. Major and minor chords, which constituted the training set, were constructed from pitches. Five pitch representations were investigated. Pitch representations generated different representations of major and minor chords. Self-
organizing map was trained five times, each time with a different chord representation. Through simulations, the network architecture, and the parameters of the training algorithm of the self-organizing map did not change.

Self-organizing map algorithms were run with SOM Toolbox 2.0 (Vesanto et al., 2000), which is a MATLAB® library developed by Laboratory of Information and Computer Science in Helsinki University of Technology. Training data were assembled with Midi Toolbox (Eerola & Toiviainen, 2004), which is also a MATLAB® library developed by Department of Music Laboratory in University of Jyvaskyla.

Self-organizing map was trained with vectors of 12 major and 12 minor chords. Chordal representations were constructed by the addition of the pitch vectors that compose the chord. For example, C major is composed of C, E, and G. The vector that represented C major chord, $\mathbf{c}_{maj}^2$, thus equals to $\mathbf{c}+\mathbf{e}+\mathbf{g}$, which were the vectors that represented pitches C, E, and G respectively. In this study, effects of input representation on the performance of self-organizing chord schema were investigated. Through simulations pitch representations were changed. Pitch representations reflected the similarity between pitches. Distance between pitch vectors reflected the similarity between them.

Many geometric models were introduced to describe similarity between pitches (Longuet-Higgins & Steedman, 1971; Shepard, 1982; Krumhansl, 2005). In this study, effects of five types of pitch representations were tested. These were orthogonal representation, the chroma circle, the circle of fifths, the torodial representation, and the lattice representation of pitches. Chroma circle captures cyclic relation between pitch categories (Shepard, 1964). Circle of fifths (Figure 1) captures the stability of pitches after a tonal context (Krumhansl & Shepard, 1979). Shepard (1982) suggested a torodial representation of pitches to capture

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2 In this article lower case bold letters denote vectors. Major and minor chords are labeled with “_maj” and “_min” suffixes.

3 Distance between vectors $\mathbf{a}$ and $\mathbf{b}$ is $|\mathbf{a}−\mathbf{b}| = \Sigma_n \sqrt{(a_i-b_i)^2}$, where $\sqrt{ }$ is the square root function, $n$ is the number of elements of vectors, and $a_i$ is the $i^{th}$ element of the vector $\mathbf{a}$. For example, if C is similar to E compared to F#, then $|\mathbf{c}−\mathbf{f#}| > |\mathbf{c}−\mathbf{e}|$; that is distance between $\mathbf{c}$ and $\mathbf{f#}$ is bigger than between $\mathbf{c}$ and $\mathbf{e}$.

4 In the previous chapters, circle of fifths was introduced as a geometric description of the harmonic relation between chords. This is not a contradiction. There are three readings of the circle of fifths. If the letters on the circle are interpreted as pitches, a continuous region on the circle covers pitches that belong to the same key. If they interpreted as major chords, and major keys, distance on the circle reflects the harmonic relation between chords and keys.
features of chroma circle and circle of fifths together. Longuet-Higgins and Steedman (1971) located pitches on a lattice in their pitch spelling algorithm (see Figure 3). The lattice is two dimensional. On the horizontal axis pitches perfect fifth apart were located nearby, reminding the circle of fifths. On the vertical axis pitches major third apart were located nearby. This representation was suggested as a model of intonation by Euler (1739, cited in Lerdahl, 2001).

Figure 3. Lattice of pitches developed by Euler (1739), and utilized by Longuet-Higgins and Steedman (1971) and Chew (2001).

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In this study, effects of input representation on the performance of self-organizing chord schema were investigated. Self-organizing chord schema was trained with chords. Chords were constructed by the addition of the pitch representations that compose the chord. Different pitch representations reflected different similarity structures between pitches. Training was repeated by changing the pitch representations. Five types of pitch representations were tested: "orthogonal", "chroma", "circle of fifths", "chroma + circle of fifths", "major3rd + circle of fifths".

In "orthogonal" representation distance between pitch vectors was equal. Figure 4 presents some pitch vectors of the “orthogonal” representation. Figure 5 presents the distance between c and the other pitch vectors of the “orthogonal” representation.

![Figure 4. Some of the pitch vectors of the "orthogonal" representation.](image)

![Figure 5. Distance between C and the other pitch vectors of “orthogonal" representation.](image)

In "chroma" representation distance between pitch vectors approximated the chroma circle. Chroma circle captures the cyclic relation between pitches (Shepard, 1964). Figure 6 presents some pitch vectors of the "chroma" representation. Figure 7 presents the distance between C and the other pitch vectors of the “chroma" representation.
In "circle of fifths" representation distance between pitch vectors approximated the circle of fifths (Figure 1). Circle of fifths captures relation between pitches after a tonal context. Pitches that belong to the same key form a continuous region on the circle of fifths. For example, F, C, G, D, A, E and B are located on the circle of fifths in this order; and these pitches belong to the C Major key. Figure 8 presents some pitch vectors of the "circle of fifths" representation. Figure 9 presents the distance between c and the other pitch vectors of the “circle of fifths” representation.
Figure 8. Some of the pitch vectors of the "circle of fifths" representation.

Figure 9. Distance between \( c \) and the other pitch vectors of the "circle of fifths" representation.

In "chroma + circle of fifths" representation distance between pitch vectors approximated the chroma circle and the circle of fifths at the same time. Shepard (1982) suggested a torodial representation of pitches to capture features of chroma circle and circle of fifths together. This can be visualized by imagining a double helix wrapped around a torus (see Shepard, 1982, fig. 5). Figure 10 presents some pitch vectors of the "chroma + circle of fifths" representation. Figure 11 presents the distance between \( c \) and the other pitch vectors of the "chroma + circle of fifths" representation.
1.00
2.00
3.00
4.00
5.00

C
C♯(Db)
D
D♯(Eb)
E
F
F♯(Gb)

Figure 10. Some of the pitch vectors of the "chroma + circle of fifths" representation.

Figure 11. Distance between c and the other pitch vectors of "chroma + circle of fifths" representation.

In "major3rd + circle of fifths" representation the distance between pitch vectors approximated distances on the lattice of pitches (Figure 3). Longuet-Higgins and Steedman (1971) utilized the lattice in their pitch spelling algorithm. Figure 12 presents some pitch vectors of the "major3rd + circle of fifths" representation. Figure 13 presents the distance between c and the other pitch vectors of the "major3rd + circle of fifths" representation.

---

### The Major3rd Rep The Circle of Fifths Rep

<table>
<thead>
<tr>
<th>Pitch Category</th>
<th>Vector Distance from C</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>[1 1 0 0 1 1 1 1 1 1 0 0 0 0 0 0]</td>
</tr>
<tr>
<td>C#(Db)</td>
<td>[1 0 0 1 1 0 0 0 0 0 1 1 1 1 1 1]</td>
</tr>
<tr>
<td>D</td>
<td>[0 0 1 1 0 0 1 1 1 1 1 1 0 0 0 0]</td>
</tr>
<tr>
<td>D#(Eb)</td>
<td>[0 1 1 0 1 1 1 0 0 0 0 0 1 1 1 1]</td>
</tr>
<tr>
<td>E</td>
<td>[1 1 0 0 0 0 0 0 1 1 1 1 1 1 1 0]</td>
</tr>
<tr>
<td>F</td>
<td>[1 0 0 1 1 1 1 1 1 0 0 0 0 0 0 1]</td>
</tr>
<tr>
<td>F#(Gb)</td>
<td>[0 0 1 1 0 0 0 0 0 0 1 1 1 1 1 1]</td>
</tr>
</tbody>
</table>

**Figure 12.** Some of the pitch vectors of the "major3rd + circle of fifths" representation.

### Distance between c and the other pitch vectors of "major3rd + circle of fifths" representation.

**Figure 13.**

Chord vectors were generated by the addition of three pitch vectors that represent the pitches of the chord. For example, C major is composed of C, E, and G. The vector that represented C major chord, \( \mathbf{c}_{maj} \), equals to \( \mathbf{c} + \mathbf{e} + \mathbf{g} \). Similarity of chords is a result of the similarity between pitches that form the chords. Figure 14 presents C major chord with different representations.
In this study, effects of input representations on the performance of self-organizing chord schema were investigated. Self-organizing map was composed of 24X36 (864) units located on a torus\(^5\). Training data consisted of 24 chord vectors (12 major and 12 minor chords). Chord vectors were normalized to guarantee a fair comparison between different representations. Training of the self-organizing map was repeated with each input representation. The network architecture and the parameters of the training algorithm were kept same. Activation of a unit on the self-organizing map reflected the cosine of the angle (correlation) between the input vector and the weight vector of the unit. Before training, weights of units were randomized. Training was divided into two phases, rough organization and fine tuning. For all simulations, average quantization error (average distance between the weight vector of the best matching unit and the corresponding input vector) was less than 0.1. Simulations repeated many times in order to be sure that average quantization error did not decrease/increase considerably. After training, 12 major and 12 minor chords were labeled on the map. Labeling was done by presenting the 12 major and 12 minor chords to the map and selecting the best matching unit (the unit with the highest level of activation) for each chord. Simulations of experimental results were conducted by reading activations of the labeled units.

Using different input representations resulted in different topographic organization of 12 major and 12 minor chords. The location of the chords on the map reflected the similarity

\(^5\) Torus is a surface having the shape of a doughnut. This network topology has been successfully applied to the self-organization of chords and keys (Leman, 1995; Leman and Carreras, 1997; Toiviainen & Krumhansl, 2003).
between representations of chords. With “orthogonal” representation, chords that share pitch(es) were located nearby on the map (Figure 15). For example, C Major shares two pitches with A minor (C and E), C minor (C and G) and E minor (E and G). The distance between C major and these chords on the map were almost equal. With the “circle of fifths” representation, the topographical organization of chords reminded the lattice of pitches (Figure 3)\(^6\). On the horizontal direction, major and minor chords that share the root (C major and C minor) alternate with major and minor chords whose roots were major third away. For example, on the one side of the C minor - C major pair, there was E minor - E major pair; on the other side there was G# minor – G# major pair. On the vertical direction rotated about 45 degrees, chords were located according to the circle of fifths. For example, C major was located near the G major on the one side, F major on the other.

\[\text{Figure 15. Topographic organization of chords with the “orthogonal” representation. Torodial surface of the map is projected on the plane. The upper and the lower edges of the figure are adjacent, as well as the left and right edges.}\]

\(^6\) Toiviainen and Krumhansl (2003) observed almost identical topographical organization from a self-organizing map trained with stability profiles of keys.
Correlations between activation of target units of self-organizing models were high (Table 1). The highest correlation was between self-organizing chord schemata that used the "circle of fifths" and the "chroma + circle of fifths" input representations ($r=0.99$, $p<.01$, $N=96$). Correlations between target unit activations of MUSACT and self-organizing models were not high, but they were significant (Table 1). The highest correlation with MUSACT was observed with the models which used the "circle of fifths" and the "chroma + circle of fifths" input representations ($r=0.43$, $p<.01$, $N=96$).

**Table 1.** Correlations between activation of target units of self-organizing models with different input representation (Maj3rd+COF, Chroma, Chroma, COF, Orthogonal) and MUSACT (Bharucha, 1987). ($N=96$, $p<.01$ for all correlations.).

<table>
<thead>
<tr>
<th></th>
<th>Maj3rd+COF</th>
<th>Chroma</th>
<th>Chroma+COF</th>
<th>COF</th>
<th>Orthogonal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chroma</td>
<td>0.82</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Chroma+COF</td>
<td>0.98</td>
<td>0.72</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COF</td>
<td>0.96</td>
<td>0.65</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orthogonal</td>
<td>0.92</td>
<td>0.90</td>
<td>0.84</td>
<td>0.79</td>
<td></td>
</tr>
<tr>
<td>MUSACT</td>
<td>0.42</td>
<td>0.31</td>
<td>0.43</td>
<td>0.43</td>
<td>0.37</td>
</tr>
</tbody>
</table>
Simulation Results and Discussion

After the training, performance of chord schemata was evaluated by simulating important findings of chord perception (Bhaucha & Krumhansl, 1983; Bharucha & Stoeckig, 1986; Bigand & Pineau, 1997; Tekman & Bharucha, 1998; Krumhansl, 1990). All of these findings were simulated successfully with the self-organizing chord schema of Tillmann et al. (2000).

Simulations were run by presenting the stimuli of the experiments as input to the map. Then activations of the target chords were read. The targets were the chords from which the human data was collected in the chosen studies. Activations of the target chords were either compared categorically with the human data or correlated with it. The temporal relation within input sequences was coded with the short-term memory model of Toiviainen and Krumhansl (2003). According to this short-term memory model, accent (importance) of a pitch is a function of duration of the pitch, and accent gradually decreases as time passes. This is calculated with the following equation (Toiviainen & Krumhansl, 2003, eq. 2).

\[
op_k = -\frac{p_k}{\tau_p} + \sum_{i=1}^{n_k} \delta(t - t_{ki}^n) - p_k \sum_{i=1}^{n_k} \delta(t - t_{ki}^f) \tag{5}\]

In this equation \( p_k \) is the time derivative of \( k \)th pitch category \( p_k \); \( \delta \) is the Dirac delta function (unit impulse function); \( \tau_p \) is time constant set to 0.5 seconds. Onset and offset times of tones having \( k \)th pitch are denoted by \( t_{ki}^n \) and \( t_{ki}^f \) respectively. \( n_k \) represents total number of times the \( k \)th pitch occurs. With equation 5 pitch accents "... saturates at about 1 s after tone onset approximating the durational accent as a function of tone duration (Parncutt 1994)." (Toiviainen & Krumhansl, 2003, p. 11).

Durational accents were integrated into memory, which was subject to a decay (Toiviainen & Krumhansl, 2003, eq. 3)

\[\frac{o}{m} = \frac{p - m}{\tau_m} \tag{6}\]

where \( \tau_m \) is set to 3 seconds which estimated capacity of auditory sensory memory (Toiviainen & Krumhansl, 2003, p. 11). These functions are available in Midi Toolbox (Eerola & Toiviainen, 2004).
Simulation results are presented in Figures 17-25. Krumhansl (1990) observed stability of chords after major and minor musical contexts. Participants listened to chord sequences in major and minor tonalities followed by a chord. They rated how well the last chord sounded after the sequence. Ratings were given in Table 7.3 of Krumhansl (1990). To simulate the results, activations of the 12 major and 12 minor chords were read after the input stimuli (C-F-Bdim-Em-Am-Dm-G-C for the major context; and Cm-Fm-Bb-Eb-Ab-Ddim-G-C for the minor context). Activations were correlated with stability ratings of the 12 major and 12 minor chords in Table 7.3 (Krumhansl, 1990). Correlations are presented in Figure 17. Correlations were high and significant. The highest correlation was observed with the "orthogonal" representation ($r=0.8$, $p<.01$, $N=24$ for the major context; $r=0.76$, $p<.01$, $N=24$ for the minor context). The lowest correlation was observed with the "circle of fifths" representation ($r=0.67$, $p<.01$, $N=24$ for the major context; $r=0.64$, $p<.01$, $N=24$ for the minor context). But all of the correlations were higher than the correlation calculated with the target unit activations of MUSACT ($r=0.56$, $p<.01$, $N=24$ for the major context; $r=0.51$, $p<.05$, $N=24$ for the minor context) (not presented in Figure 17).

![Figure 17. Correlations between stability ratings of chords (Krumhansl, 1990, table 7.3) and targets unit activations. (N=24, p<.01 for all correlations.)](image)

Bhaucha and Krumhansl (1983) investigated perception of chordal similarity. Participants listened to chord sequences in five different keys (C, G, A, B, and F# major) followed by all possible pairs of chords from the C and F# major keys. Participants rated the similarity of the
chord pairs. To explain the data, principles of contextual distance and contextual asymmetry were articulated (Bhaucha & Krumhans, 1983; Krumhansl, 1990). The principle of contextual distance states that the higher the two chords are in the hierarchy of stability, the similar they were. The principle of contextual asymmetry states that the last chord of the pair contributed the perceived similarity more, compared to the first one.

To simulate the results, experimental stimuli (IV, V, I cadence in C, G, A, B, and F# major keys) followed by the first chord of the pair was presented as input, and the activation of the second chord of the pair was read. The average activation of the second chord was calculated for four different types of pairs: when both chords in C major, both chords in F# major, the first in C major and the second in F# major, the first in F# major and the second in C major. For brevity, simulation results of the “orthogonal” and the “circle of fifths” representations were presented only. Performances of the other self-organizing maps were not significantly different. Figures 18 and 19 present the simulation results of the contextual distance principle with the “orthogonal” and the “circle of fifths” input representations respectively. When the chords of the pair were in C major, the average activation of the second chord decreases as the distance between the context and the C major increases on the circle of fifths; when the chords of the pair were in F# major, the average activation of the second chord increases as the distance between the context and the F# major increases on the circle of fifths.

The contextual asymmetry principle was also simulated with the “orthogonal” and the “circle of fifths” input representations (Figures 20 and 21). When the chords of the pair were not in the same key (i.e. one is in C major and the other is in F# major) the average activation of the second chord decreases as distance between the context and key of the of the second chord increases on the circle of fifths.

---

7 Performance of the other self-organizing maps was similar.
Figure 18. Average activation of the second chord of the pair on the model trained with the “orthogonal” input representation, after the stimuli of Bhaucha and Krumhansl (1983) and the first chord of the pair as input. Chords of the pair were in the same key.

Figure 19. Average activation of the second chord of the pair on the model trained with the “circle of fifths” input representation, after the stimuli of Bhaucha and Krumhansl (1983) and the first chord of the pair as input. Chords of the pair were in the same key.
Figure 20. Average activation of the second chord of the pair on the model trained with the “orthogonal” input representation, after the stimuli of Bhaucha and Krumhansl (1983) and the first chord of the pair as input. Chords of the pair were in different keys.

Figure 21. Average activation of the second chord of the pair on the model trained with the “circle of fifths” input representation, after the stimuli of Bhaucha and Krumhansl (1983) and the first chord of the pair as input. Chords of the pair were in different keys.
Table 8.2 of Krumhansl (1990) presented data of the diatonic pairs from the study of Bharucha and Krumhansl (1983). Activations of the second chord were correlated with the ratings of Table 8.2 (Krumhansl, 1990). Correlations are presented in Figure 22. Correlations were high and significant. The highest correlation was observed with the "circle of fifths" input representation (r=0.86, p<.01, N=36). The lowest correlation was observed with "chroma" input representation (r=0.35, p<.05, N=36). But all of the correlations were lower than the correlation calculated with the target unit activations of MUSACT (r=0.97, p<.01, N=36) (not presented in Figure 22).

Bharucha and Stoeckig (1986) demonstrated the facilitative performance on the target chord when the prime was closer to the target on the circle of fifths. This study was the first demonstration of the priming paradigm and its relation with the circle of fifths. In order to simulate this result, activation of the target unit should be higher after the input one-step-away from the target, compared to the input six-steps-away. All of the models with different input representations simulated the result (Figure 23).
Tekman and Bharucha (1998) demonstrated the facilitative performance on a target chord when the prime was two-steps away from the target on the circle of fifths compared to when it was four-steps away. Target and four-steps-away prime share a pitch. Target and two-steps-away prime do not share a pitch. This study observed the fact that priming was not due to the shared pitches (not due to the acoustical similarity), but due to the distance on the circle of fifths (harmonic similarity). In order to simulate the result of Tekman and Bharucha (1998), activation of the target unit must be higher after the input two-steps-away from the target, compared to the input four-steps-away. Only models with the "circle of fifths" and the "chroma + circle of fifths" input representations simulated the result (Figure 24). Other input representations failed to simulate this finding.

Bigand and Pineau (1997) demonstrated the facilitative performance on a target chord when it was tonic compared to when it was subdominant. This study observed the fact that harmonic function of a chord affects the priming of the chord. In order to simulate this result, average activation of the tonic target unit should be higher, compared to the average activation of the subdominant target unit. All input representations simulated the result (Figure 25).
As a result, training self-organizing networks with different input representations gave promising results for modeling of chord perception with a bottom-up model. This model is architecturally dissimilar to MUSACT (Bharucha, 1987), which proposes that top-down and bottom-up associations between representations of chords and keys shapes the mental representation of chords. Tillmann et al. (2000) simulated the self-organization of a tonal schema, which is architecturally identical to MUSACT (Bharucha, 1987). To reach this end,
they pruned and scaled weights, and changed the activation and output functions of the units. On the other hand, simulations of the current study showed that top-down connections were not necessary to model perception of chords. Important findings of chord perception were modeled with self-organizing networks that have bottom-up connections only. Input representation was critical for a successful simulation. Different input representations simulated many of the findings, but not all of them. All of the findings were simulated only when the self-organizing map utilized the "circle of fifths" and the "chroma + circle of fifths" input representations.

In addition, the highest correlation between the activation patterns of MUSACT (Bharucha, 1987) and the self-organizing bottom up models was observed with the "circle of fifths" and the "chroma + circle of fifths" input representations. Correlation of chord stability ratings with the target unit activations of these models were higher compared to the correlation with the target unit activations of MUSACT (Bharucha, 1987). However, MUSACT (Bharucha, 1987) was better correlated with the similarity ratings of diatonic chords. This result suggest that relations between diatonic chords were better captured with MUSACT; but stability ratings were better captured with the bottom-up models with the "circle of fifths" and the "chroma + circle of fifths" input representations.

On the other hand, it may be proposed that none of the models simulated the results of Tekman and Bharucha (1998). In Tekman and Bharucha (1998), there were four different prime durations: 50, 500, 1,500, and 2,500 ms. When the prime played for 50 ms, participants responded faster after the four-steps-away prime (the acoustically similar prime) compared to the two-steps-away prime (the harmonically related prime). For durations of 500 ms or more, this pattern reversed. This suggests that through time, the nature of priming changes from the acoustical similarity to the harmonic relatedness. MUSACT, as well as the model of Tillmann et al. (2000), simulated the result of 50 and 500 ms or longer primes, by reading target unit activations at the beginning and the end of the equilibrium. The model develop in this study doesn’t have the mechanisms to simulate the time course of chord priming. Signals reach instantly from the pitch representations to the chord representations. Therefore, only the result with 500 ms or longer primes was simulated in this study. The simulations with MUSACT and the model of Tillmann et al. (2000) were carried by reading the target chord unit activation. In other words they assumed that priming with 50 ms stimuli
is also governed by the mental representations of chords but not mental representations of pitches or sensory representations of sounds.

This study showed that it is possible to simulate important findings of chord perception with self-organizing bottom-up models. In these models the activation of chords are not shaped by the activation of keys. Only the representation and activation of pitches determine the chordal activation. It is critical to select proper pitch representation for a valid bottom-up network. This study can be seen as another theory of chord perception. According to this theory, mental representations of chords are not shaped by the mental representations of keys; i.e. there is no top-down influence on chordal percepts. One way to test this theory is to contrast the predictions of interactive and bottom-up models; and conduct empirical research according to the predictions.

Recent studies showed that MUSACT (Bharucha, 1987) failed to predict effects of non-diatonic chords on chord priming (Atalay, Tekman, 2006; Atalay, Tekman, Toiviainen, 2006; see also Atalay, 2007). These findings suggested that listeners of Western music internalized not only the common harmonic relations between chords (those described with the circle of fifths), but also chordal relations that are contradictory to the general rules. The common and the exceptional chordal relations cannot be represented within the same interactive activation network, where activation of a unit is partly determined as a result of reverberation of the activation within the whole network. Bottom-up networks can represent contradictory relations within the same network since activation is a result of local associations. Indeed Rumelhart and McClelland (1986) showed that bottom-up networks with error-back-propagation training algorithm could learn both general rules and exceptions of past tense formation in English. Studying bottom-up models seems to be important to develop comprehensive models of chord perception.

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References


